

Short Range Structure in the $X(3872)$

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It is proposed that the newly discovered $X(3872)$ is a $J^{PC} = 1^{++} D^0 \bar{D}^{0*}$ hadronic resonance stabilized by admixtures of $\omega J/\psi$ and $\rho J/\psi$. A specific model of the state is constructed and tests of its internal structure are suggested via the predicted decay modes $D^0 \bar{D}^0 \pi^0$, $D^0 \bar{D}^0 \gamma$, $\pi^+ \pi^- J/\psi$, and $\pi^+ \pi^- \pi^0 J/\psi$.

I. INTRODUCTION

The Belle collaboration has recently announced[1] the discovery of a resonance, $X(3872)$, in the $\pi^+ \pi^- J/\psi$ subsystem of the process

$$B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi \quad (1)$$

at a mass of $3872.0 \pm 0.6 \pm 0.5$ MeV and with a width

$$\Gamma < 2.3 \text{ MeV} \quad (95\% \text{ C.L.}). \quad (2)$$

This state, which has been confirmed by the CDF collaboration[2], has attracted some attention because of its unusual properties. Specifically, the state appears to be too heavy to be a 1D charmonium state and too light to be 2P charmonium or a $c\bar{c}$ hybrid. See Ref. [3] for a detailed assessment of possible charmonium assignments and decay modes.

Alternatively, the proximity of the state to $D\bar{D}^*$ threshold strongly suggests that the X may be a weakly bound $D\bar{D}^*$ resonance[4, 5, 6, 7, 8], sometimes called a mesonic ‘molecule’ or a ‘deuson’[9]. This is an old idea which has been applied to a variety of mesons with unusual characteristics such as the $\psi(4040)$ [10], $f_1(1420)$ [11, 12], $\eta(1440)$ [13], $f_J(1720)$ [14], $a_0(980)$, and $f_0(975)$ [12, 15].

In this note I assume that the $X(3872)$ is indeed a $D\bar{D}^*$ resonance and present a detailed analysis of its expected properties based on a simple model of quark interactions. This model incorporates the nonrelativistic quark model with additional dynamics due to pion exchange. The idea is to capture the predictive power of a microscopic formalism of short range quark dynamics along with the important long range dynamics mediated by pion exchange processes. Versions of this idea have been applied to baryon-baryon interactions since the 1980’s[16], where, of course, pion exchange is of fundamental importance; another variant has recently enjoyed some vogue in baryon physics[17].

It is natural to expect that the putative $D\bar{D}^*$ bound state is in a relative S-wave since this is typically where inter-hadron forces are strongest. In this case pion-mediated interactions (see below) favour the isoscalar channel, which in turn implies a $J^{PC} = 1^{++}$ state. I will therefore henceforth refer to the bound state interpretation of the $X(3872)$ as the $\hat{\chi}_{c1}(3872)$. The remainder of this paper focusses on the properties of this state.

Although pion exchange forces dominate the structure of the $\hat{\chi}_{c1}$ (in analogy to the deuteron), short range quark dynamics are present and assist in binding the $\hat{\chi}_{c1}$ via mixing to hidden charm vector- J/ψ states. Indeed, $\omega J/\psi$ and $\rho J/\psi$ are very nearly degenerate with $D\bar{D}^*$ and one must expect some admixture of these states – an effect which will be strongly enhanced by the near-zero energy denominator. Such mixing is also important in driving possible decay modes of the $\hat{\chi}_{c1}$ and is therefore central to determining its properties. Finally, the binding energy of the $\hat{\chi}_{c1}$ is comparable to mass differences in the available charge channels and one can expect strong isospin violating effects in this resonance. This heretofore unexplored dynamics is thoroughly examined in the following. Detailed predictions of binding energies and branching fractions are presented along with possible experimental tests of $\hat{\chi}_{c1}$ structure.

II. $D\bar{D}^*$ DYNAMICS

Long range pion exchange effects are expected to dominate the physics of a weakly bound state such as the $\hat{\chi}_{c1}$. Nevertheless, as discussed above, short range quark interactions can give rise to important mixing effects. We therefore consider a model which appends pion exchange dynamics to the nonrelativistic quark model. The model is used to extract effective interactions for $D\bar{D}^* - D\bar{D}^*$, $D\bar{D}^* - \omega J/\psi$, and $D\bar{D}^* - \rho J/\psi$ scattering. These interactions are then

employed in a nonrelativistic coupled channel Schrödinger equation to extract bound state properties (one expects the nonrelativistic formalism to be accurate for weakly bound states of relatively massive components as is the case with the $\hat{\chi}_{c1}$).

A. Quark Exchange Induced Effective Interaction

The quark model employed here assumes nonrelativistic quark dynamics mediated by an instantaneous confining interaction and a short range spin-dependent interaction motivated by one gluon exchange. The colour structure is taken to be the quadratic form of perturbation theory. This is an important assumption for multiquark dynamics which has received support from recent lattice computations for both confinement[18] and multiquark interactions[19]. The final form of the interaction is thus taken to be

$$\sum_{i<j} \frac{\lambda(i)}{2} \cdot \frac{\lambda(j)}{2} \left\{ \frac{\alpha_s}{r_{ij}} - \frac{3}{4} b r_{ij} - \frac{8\pi\alpha_s}{3m_i m_j} \mathbf{S}_i \cdot \mathbf{S}_j \left(\frac{\sigma^3}{\pi^{3/2}} \right) e^{-\sigma^2 r_{ij}^2} \right\}, \quad (3)$$

where λ is a colour Gell-Mann matrix, α_s is the strong coupling constant, b is the string tension, m_i and m_j are the interacting quark or antiquark masses, and σ is a range parameter in a regulated spin-spin hyperfine interaction. The parameters used were $\alpha_s = 0.59$, $b = 0.162 \text{ GeV}^2$, $\sigma = 0.9 \text{ GeV}$, and 0.335, 0.55, and 1.6 GeV for up, strange, and charm quark masses respectively. Relevant meson masses obtained from this model are $\rho = 0.773 \text{ GeV}$, $J/\psi = 3.076 \text{ GeV}$, $D = 1.869 \text{ GeV}$, and $D^* = 2.018 \text{ GeV}$, in good agreement with experiment.

Meson-meson interactions are obtained by computing the Born order scattering amplitude for a given process[12, 20]. Because of the colour factors in Eq. 3 this amplitude necessarily involves an exchange of quarks between the interacting mesons. Thus the leading order $D\bar{D}^*$ interaction couples $D\bar{D}^*$ with hidden charm states such as $\rho J/\psi$ and $\omega J/\psi$. This amplitude may be unitarized by extracting an effective potential and iterating it in a Schrödinger equation[12]. The method has been successfully applied to a variety of processes such as KN scattering[21] and J/ψ reactions relevant to RHIC physics[22]. It has even proven surprisingly useful for relativistic (and chiral) reactions such as $\pi\pi$ scattering[12, 20].

The S-wave Born order scattering amplitude for $D\bar{D}^* - \omega J/\psi$ scattering is shown in Fig. 1. Here $D\bar{D}^*$ refers to the isoscalar positive charge parity state $1/\sqrt{2}(D\bar{D}^* + \bar{D}D^*)_S^0$. The scattering amplitude is dominated by the confinement interaction of Eq. 3 (this is in contrast to light meson scattering which is dominated by the hyperfine interaction). An effective potential is extracted by equating the scattering amplitude to that obtained for point-like mesons interacting via an arbitrary S-wave potential. It is convenient to parameterize this potential as a sum of gaussians:

$$V_q = \sum_i a_i e^{-r^2/2b_i^2}. \quad (4)$$

The fit to the quark level amplitude is illustrated in Fig. 1 (left panel) and the resulting potential is shown in Fig. 1 (right panel). The distinctive “mermaid potential” seen here is due to destructive interference between diagrams in the quark level amplitude. Thus details of the potential are sensitive to the assumed microscopic interaction, however, its general shape and strength are quite robust[12].

The derived parameters of Eq. 4 were $a_1 = 6.35 \text{ GeV}$, $b_1 = 1.166 \text{ GeV}^{-1}$ and $a_2 = -6.82 \text{ GeV}$, $b_2 = 1.096 \text{ GeV}^{-1}$. These parameters were obtained for the confinement portion of the “prior” form of the scattering amplitude. The “post” form yields $a_1 = 3.82 \text{ GeV}$, $b_1 = 1.20 \text{ GeV}^{-1}$, $a_2 = -4.21 \text{ GeV}$, and $b_2 = 1.125 \text{ GeV}^{-1}$. Post and prior forms of a scattering amplitude refer to different schemes for constructing the time evolution operator which exist in the scattering of composite systems. In principle these give rise to the same scattering amplitude, but approximations and inaccurate wavefunctions can cause slight differences as indicated in the figure. We employ the average potential indicated by the solid line of Fig. 1 (right) in the following. Finally, the isovector $D\bar{D}^* - \rho J/\psi$ effective potential is identical to its isoscalar analogue.

The mermaid form of the effective potential implies that quark exchange effects can cause binding in the coupled $D\bar{D}^*$, $\omega J/\psi$ or $\rho J/\psi$ systems; however, direct computations indicate that the potential depth is not sufficient to form a resonance. We therefore turn to an examination of pion exchange induced dynamics in the $D\bar{D}^*$ system.

B. Pion Exchange Induced Effective Interaction

I choose to follow the method of Törnqvist[13] in constructing an effective pion-induced interaction. This is based on a microscopic quark-pion interaction familiar from nuclear physics:

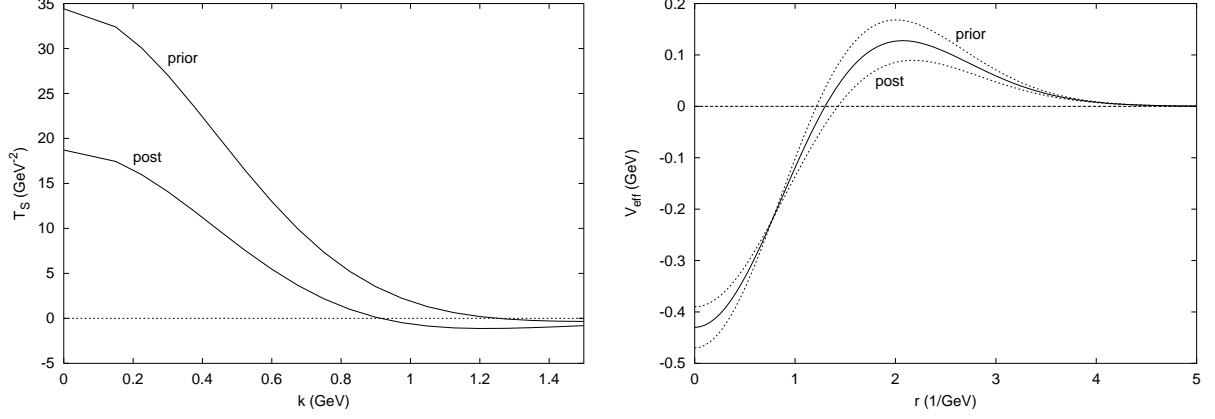


FIG. 1: (left) S-Wave Scattering Amplitude for $DD^* \rightarrow \omega J/\psi$. (right) Effective Potential for $DD^* \rightarrow \omega J/\psi$

$$\mathcal{L} = \frac{g}{\sqrt{2}f_\pi} \int d^4x \bar{\psi}(x) \gamma^\mu \gamma_5 \tau^a \psi(x) \partial_\mu \pi^a(x). \quad (5)$$

Here $f_\pi = 92$ MeV is the pion decay constant, τ is an SU(2) flavour generator, and g is a coupling to be determined. The effective potential is derived by projecting the quark level interactions onto hadronic states in the nonrelativistic limit. In the case of pseudoscalar-vector states one obtains[13]

$$V_\pi = -\gamma V_0 \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} C(r) + \begin{pmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix} T(r) \right] \quad (6)$$

where

$$C(r) = \frac{\mu^2}{m_\pi^2} \frac{e^{-\mu r}}{m_\pi r}, \quad (7)$$

$$T(r) = C(r) \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right), \quad (8)$$

and

$$V_0 \equiv \frac{m_\pi^3}{24\pi} \frac{g^2}{f_\pi^2} \approx 1.3 \text{ MeV}. \quad (9)$$

The matrix elements refer to S- and D-wave components of the pseudoscalar-vector state in analogy with the deuteron. The strength of the interaction has been fixed by comparing to the πNN coupling constant via the relationship $g_{\pi NN}^2/4\pi = 25/18 \cdot m_\pi^2 g^2/f_\pi^2$. This allows a prediction of the D^* decay width which is in good agreement with experiment[13]. The parameter μ is typically the pion mass, however, one can incorporate recoil effects in the potential by setting $\mu^2 = m_\pi^2 - (m_V - m_{pS})^2$. The results presented here are insensitive to the value of μ and I take $\mu = 130$ MeV in the following. Finally, the coupling γ is a spin-flavour matrix element which takes on the following values: $\gamma = 3$ for $I = 0$, $C = +$; $\gamma = 1$ for $I = 1$, $C = -$; $\gamma = -1$ for $I = 1$, $C = +$; and $\gamma = -3$ for $I = 0$, $C = -$. Thus the isoscalar positive charge parity channel is the most likely to form bound states and subsequent discussion focusses on it.

The potential of Eq. 8 is an illegal quantum mechanical operator and must be regulated, typically with a dipole form factor. The regulator scale, Λ may be fixed by comparison with nuclear physics; for example NN interactions yield preferred values for Λ in the range 0.8 GeV to 1.5 GeV depending on model details. Alternatively, reproducing the deuteron binding energy requires $\Lambda \approx 0.8$ GeV. Törnqvist has employed an intermediate value of $\Lambda = 1.2$ GeV which is appropriate for D mesons and this is taken as the canonical cutoff in the following.

Integrating the coupled S/D wave system for the $1^{++} B\bar{B}^*$ system yields a bound state of mass 10562 MeV, in agreement with Ref. [13]. Similarly a $0^{-+} B\bar{B}^*$ bound state of mass 10545 MeV arises from this formalism.

Unfortunately, D mesons are sufficiently light that the $D\bar{D}^*$ system does not bind with canonical parameters. However, the combined pion and quark induced effective interactions are sufficient to cause binding. The properties of this bound state are explored in the next section.

III. PROPERTIES OF THE $\hat{\chi}_{c1}(3872)$

The proceeding considerations indicate that the isoscalar positive charge conjugation sector is the most likely to bind in the $D\bar{D}^*$ system. Furthermore the small branching fraction of the $\hat{\chi}_{c1}$ to $\pi\pi J/\psi$ implies a small isovector $\rho J/\psi$ component in the $\hat{\chi}_{c1}$ wavefunction. Thus a good initial study is provided by the coupled channel $1/\sqrt{2}(D\bar{D}^* + \bar{D}D^*)_S^0$, $1/\sqrt{2}(D\bar{D}^* + \bar{D}D^*)_D^0$, and $\omega J/\psi$ system. Utilizing the potentials of Eqs. 4 and 6 and meson masses of $D = 1.869$ GeV, $D^* = 2.01$ GeV, $\omega = 0.78$ GeV, and $J/\psi = 3.1$ GeV yields a single bound state of mass 3.872 GeV without adjusting any parameters, in remarkable agreement with the mass of the X . The $\hat{\chi}_{c1}$ wavefunction is plotted in Fig. 2 (left panel); one sees typical deuteron-like wavefunctions with strong D-wave and $\omega J/\psi$ components. All three components are required to achieve binding for this state.

Although encouraging, this result must not be taken too seriously because the binding energy is comparable to the difference in energies of the various relevant charge channels. Thus isospin breaking effects are expected to be important and must be incorporated in the formalism. This is achieved by including isovector channels and allowing for differing thresholds. Restricting attention to nearby vector meson - J/ψ states and neglecting the coupling to charmonium states[24] yields six possible channels:

- $\rho J/\psi$ at 3.8679 GeV
- $\frac{1}{\sqrt{2}}(D^0\bar{D}^{0*} + \bar{D}^0D^{0*})_S$ at 3.8712 GeV
- $\frac{1}{\sqrt{2}}(D^0\bar{D}^{0*} + \bar{D}^0D^{0*})_D$ at 3.8712 GeV
- $\frac{1}{\sqrt{2}}(D^+D^{-*} + D^-D^{+*})_S$ at 3.8793 GeV
- $\frac{1}{\sqrt{2}}(D^+D^{-*} + D^-D^{+*})_D$ at 3.8793 GeV
- $\omega J/\psi$ at 3.8795 GeV.

One sees an immediate problem: the threshold for $\rho J/\psi$ is too low to allow a resonance at 3.872 ± 1 GeV. However, the mass of the ρ is rather poorly defined due to its large width and some leeway in fixing threshold for this channel is permissible. I therefore adopt the simple prescription of setting the ρ mass equal to that of the ω at 0.7826 GeV. Varying this prescription made negligible changes to the following results. The quark level coupling of $(D\bar{D}^*)_D$ states to S-wave ρ - or ω - J/ψ states is small and is neglected. The final step is to form effective interactions from appropriate combinations of isospin basis interactions. For example the $D^0\bar{D}^{0*} - \bar{D}^0D^{0*}$ interaction is given by Eq. 6 with $\gamma = 1$. The resulting numerical six channel problem must be studied with some care because the binding energies are small relative to the natural scales of the system.

It is of interest to study the properties of possible bound states as a function of their binding energy. This has been achieved by allowing the regulator scale to vary between 1.2 and 2.3 GeV. Binding is seen to occur for Λ larger than approximately 1.45 GeV. Wavefunction coefficients (defined as $\int |\varphi_\alpha|^2$ where α is a channel index) are shown as a function of binding energy in Fig. 2 (right panel). It is clear that the $D^0\bar{D}^{0*}$ component dominates the wavefunction, especially near threshold. However, the D^+D^{-*} component rises rapidly in strength with isospin symmetry being recovered at surprisingly small binding energies (on the order of 30 MeV). Alternatively, the $\omega J/\psi$ component peaks at roughly 17% at $E_B \approx 9$ MeV. The contribution of the $\rho J/\psi$ wavefunction remains small, peaking at less than 1% very close to threshold.

It is possible to estimate decay rates in a simple fashion once an explicit wavefunction is known. This is because the small binding energy of the $\hat{\chi}_{c1}$ implies that its constituent particles are nearly on-shell and therefore $\hat{\chi}_{c1}$ decay amplitudes are well approximated by constituent decay amplitudes. Thus, for example, the $\pi^+\pi^-J/\psi$ decay mode arises predominantly from the $\rho J/\psi$ wavefunction component. The channel strengths of Fig. 2 therefore allow simple estimates of a variety of branching fractions based on the widths of the D^* , ω , and ρ (decays of the D and J/ψ mesons are neglected here but can be computed easily). The results for a variety of modes are presented in Table I as a function of the binding energy.

The broadest particle in the $\hat{\chi}_{c1}$ system is the ρ with a width of 150 MeV and it is the $\rho J/\psi$ component which has the largest branching fraction, even though it is strongly suppressed in the wavefunction. The next strongest mode is provided by the $\pi^+\pi^-\pi^0$ decay of the ω which is enhanced relative to other modes due to strong mixing

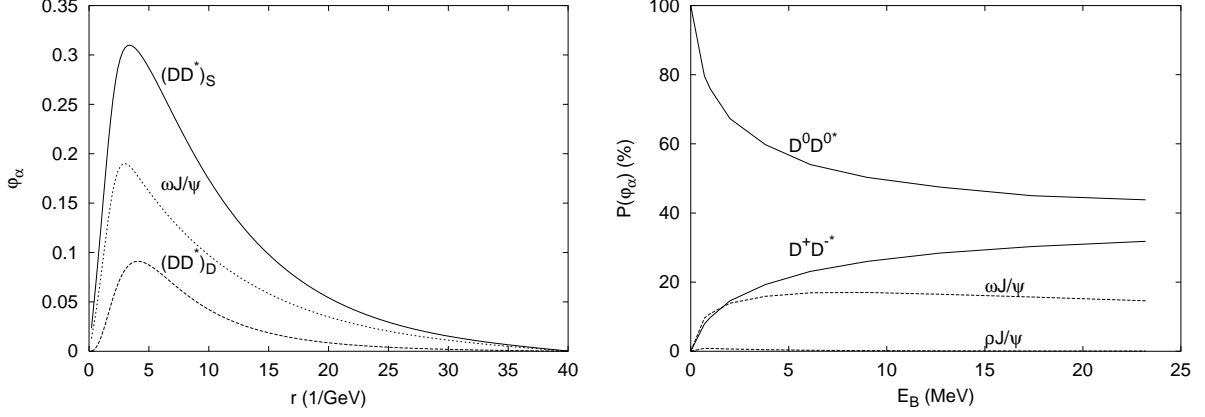


FIG. 2: (left) Three Channel Isoscalar Wavefunction Components. (right) Component Strength vs. Binding Energy.

with $\omega J/\psi$. Unfortunately, only a very rough upper limit on the total width of the D^{0*} exists[23] so estimates of the $D^0\bar{D}^0\pi^0$ and $D^0\bar{D}^0\gamma$ decay widths are essentially useless. The figures in the table have been obtained by assuming that $\Gamma(D^{0*} \rightarrow D^0\gamma) \approx 25$ keV and $\Gamma(D^{0*} \rightarrow D^0\pi^0) \approx 43$ keV; both of these estimates are anchored in $D^{\pm*}$ decays and should be reliable. Notice that the $D^\pm\pi^\mp$ mode is closed. All other possible decay modes of the $\hat{\chi}_{c1}$ are relatively small, although the $\pi^0\gamma J/\psi$ mode may be of interest if it is detectable.

TABLE I: Some Decay Modes of the $\hat{\chi}_{c1}(3872)$ (keV).

B_E (MeV)	$D^0\bar{D}^0\pi^0$	$D^0\bar{D}^0\gamma$	$D^+\bar{D}^-\pi^0$	$(D^+\bar{D}^0\pi^- + \text{c.c.})/\sqrt{2}$	$D^+\bar{D}^-\gamma$	$\pi^+\pi^-J/\psi$	$\pi^+\pi^-\gamma J/\psi$	$\pi^+\pi^-\pi^0 J/\psi$	$\pi^0\gamma J/\psi$
0.7	67	38	5.1	4.7	0.2	1290	12.9	720	70
1.0	66	36	6.4	5.8	0.3	1215	12.1	820	80
2.0	57	32	9.5	8.6	0.4	975	9.8	1040	100
3.8	52	28	12.5	11.4	0.6	690	6.9	1190	115
6.1	46	26	15.0	13.6	0.7	450	4.5	1270	120
9.0	43	24	16.9	15.3	0.8	285	2.9	1280	125
12.7	38	22	18.5	16.7	0.9	180	1.8	1240	120

IV. CONCLUSIONS

I have argued that the $X(3872)$ is a $J^{PC} = 1^{++} D\bar{D}^*$ hadronic resonance with important admixtures of $\rho J/\psi$ and $\omega J/\psi$ states, dubbed the $\hat{\chi}_{c1}$. This assertion is supported by detailed computations in a microscopic model which incorporates pion and quark exchange interactions. The model has been heavily tested on nuclear physics and meson-meson scattering data and can be regarded as reasonably reliable. The $1^{++} \hat{\chi}_{c1}$ is the only $D\bar{D}^*$ state which binds; no other J^{PC} or charge modes exist in this model. Furthermore, no $D\bar{D}$ molecules are expected. It is likely, however, that a rich $D^*\bar{D}^*$, $B\bar{B}^*$ and $B^*\bar{B}^*$ spectrum exists. Thus the discovery of the $X(3872)$ may be the entree into a new regime of hadronic physics which will offer important insight into the workings of strong QCD and should help clarify many open issues in light quark spectroscopy. Indeed, the experimental and theoretical analysis of heavy molecules is simplified because of their hidden flavour components.

It is clear that further experimental studies of the $X(3872)$ are of great importance. For example, determining its spin and parity are of immediate concern. The fact that the X is polarized in $B \rightarrow KX$ will help greatly in this. Furthermore, detecting a $\pi^0\pi^0 J/\psi$ decay mode would immediately eliminate the $\hat{\chi}_{c1}$ interpretation of the X .

It is also important to gather enough events to reconstruct the invariant mass of various subsystems such as $\pi^+\pi^-$ in $\pi^+\pi^-J/\psi$ (which should peak at the ρ mass). Perhaps a more interesting test would be the invariant mass distribution of the $\pi^+\pi^-\pi^0$ subsystem in the $\pi^+\pi^-\pi^0 J/\psi$ decay mode, which should have all of its events near the edge of phase space due to the narrow width of the virtual ω . It is therefore encouraging that the $3\pi J/\psi$ decay mode is roughly 1/2 the strength of the $2\pi J/\psi$ mode. Although some events will be lost due to the decreased efficiency in detecting neutral pions, this deficit should be made up by the new data being collected at Belle and BaBar.

Lastly, determining branching fractions, especially those arising from different wavefunction components such as $D^0\bar{D}^0\pi^0$, $\pi^+\pi^-J/\psi$, and $\pi^+\pi^-\pi^0 J/\psi$, would help greatly in pinning down the internal structure of the X and provide

an intriguing glimpse into a new realm of hadronic physics.

Acknowledgments

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